B.Tech.
(SEMESTER-IV) THEORY EXAMINATION, 2012-13
MATHEMATICS – III

Time: 3 Hours  
[ Total Marks: 100 ]

Note: Attempt questions from each section as indicated. The symbols have their usual meaning.

SECTION – A

1. All parts of this question are compulsory:  
   \[10 \times 2 = 20\]

   (a) Find the constants a, b and c such that the function \( f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + y^2) \) is analytic.

   (b) Evaluate the integral \( \int_C \frac{e^{iz}}{z^3} \, dz \), where C: \( |z| = 1 \).

   (c) The first-four central moments of a distribution are 0, 2.5, 0.7 and 18.75. Comment on the kurtosis of the distribution.

   (d) The equations of two lines of regression are \( 3x + 12y = 19 \) and \( 9x + 3y = 46 \). Find the mean of \( x \) and the mean of \( y \).

   (e) Enlist the methods by which Trend values can be determined.

   (f) Find the moment generating function of Poisson distribution.

   (g) Show that \( hD = -\sinh^{-1}(\mu \delta) \).
(h) Find the value of $\Delta^2(ab^c)$.

(i) Show that $y' = \frac{1}{h} \left[ \Delta y - \frac{1}{2} \Delta^2 y + \frac{1}{3} \Delta^3 y - \frac{1}{4} \Delta^4 y + \ldots \right]$.

(j) Calculate the value of $\int_4^{5.2} \log_e x \, dx$ by Trapezoidal rule.

SECTION – B

2. Attempt any three parts:

(a) Using the method of contour integration, evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$.

(b) Find the multiple linear regression of $x_1$ on $x_2$ and $x_3$ from the data relating to three variables:

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>9</th>
<th>13</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>15</td>
<td>12</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$x_3$</td>
<td>30</td>
<td>24</td>
<td>20</td>
<td>14</td>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

(c) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

(d) Perform four iterations of the Newton-Raphson method to obtain the approximate value of $(17)^{\frac{1}{3}}$ starting with initial approximation $x_0 = 2$.

(e) Find the value of $y(1.1)$, using Runge-kutta method of fourth order, given that $\frac{dy}{dx} = y^2 + xy$, $y(1) = 1.0$, take $h = 0.05$. 

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3. (a) Using Cauchy’s integral formula, evaluate
\[ \int_\mathcal{C} \frac{\sin \pi z^2 + \cos \pi z^2}{(z - 1)(z - 3)} \, dz \]
where \( \mathcal{C} : |z| = 2 \).

(b) Prove that \( \cosh \left( z + \frac{1}{z} \right) = a_0 + \sum_{n=1}^{\infty} a_n \left( z^n + \frac{1}{z^n} \right) \),
where \( a_n = \frac{1}{2\pi} \int_0^{2\pi} \cos n\theta \cdot \cosh (2\cos \theta) \, d\theta \).

(c) State and prove Cauchy’s Residue Theorem.

4. (a) Find the least squares fit of the form \( y = a + bx^2 \) to the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

(b) Show that the regression co-efficients are independent of the change of origin but not of scale.

(c) Find the moment generating function for triangular distribution defined by

\[ f(x) = \begin{cases} 
  x, & 0 \leq x \leq 1 \\
  2-x, & 1 \leq x \leq 2 
\end{cases} \]

5. (a) If the variance of the Poisson distribution is 2, find the probabilities for \( r = 1, 2, 3 \)
and 4 from the recurrence relation of the Poisson distribution. Also find \( P(r \geq 4) \).

(b) Given the following information in the usual notations:

\( n_1 = 7, n_2 = 6, S_1^2 = 6.21, S_2^2 = 5.23, \bar{x} = 30 \) and \( \bar{y} = 28 \).

Test the hypothesis that the two samples have come from population having equal means.

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P.T.O.
(c) 100 students of an engineering institute obtained the following grades in Mathematics paper:

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>15</td>
<td>17</td>
<td>30</td>
<td>22</td>
<td>16</td>
<td>100</td>
</tr>
</tbody>
</table>

Using $\chi^2$–test, examine the hypothesis that the distribution of grades is uniform.

6. (a) Find the missing term in the table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>45.0</td>
<td>49.2</td>
<td>54.1</td>
<td>?</td>
<td>67.4</td>
</tr>
</tbody>
</table>

(b) Show that the Regula-Falsi Method has linear rate of convergence.

(c) Given the data $f(1) = 4$, $f(2) = 5$, $f(7) = 5$, $f(8) = 4$. Find the value of $f(6)$ and also the value of $x$ for which $f(x)$ is maximum or minimum.

7. (a) Find the derivative of $f(x)$ at $x = 0.4$ from the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1.10517</td>
<td>1.22140</td>
<td>1.34986</td>
<td>1.49182</td>
</tr>
</tbody>
</table>

(b) Use Picard’s method to approximate the value of $y$ when $x = 0.1$ given that $y=1$ when $x = 0$ and $\frac{dy}{dx} = 3x + y^2$.

(c) Solve the system:

\[ x_1 + x_2 + x_3 = 1, \]
\[ 3x_1 + x_2 - 3x_3 = 5, \]
\[ x_1 - 2x_2 - 5x_3 = 10 \]

by Crout’s method.