B.Tech
(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10
MATHEMATICS -III

Time : 3 Hours] [Total Marks : 100

Note : Attempt all questions. Every question carries equal marks.

1. Attempt any two of the following : 10×2=20

(a) Find the Fourier transform of

\[ f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases} \]

Using this result evaluate

\[ \int_{-\infty}^{\infty} \frac{\sin at \cos at}{t} \, dt. \]

(b) State and prove the convolution theorem for the Fourier transform. Verify this theorem for the functions

\[ f(t) = e^{-t} \text{ and } g(t) = \sin t. \]

(c) Define the Z-transform. Solve the difference equation

\[ y_{n+2} + 6y_{n+1} + 9y_n = 2^n \text{ with } y_0 = y_1 = 0. \]
Attempt any two of the following:

(a) If \( f(z) = u + iv \) is an analytic function of \( z = x + iy \) and \( u - v = e^{-x} \left[ (x - y) \sin y - (x + y) \cos y \right] \), then find \( u, v \) and the analytic function \( f(z) \).

(b) State and prove the Cauchy's integral theorem for the derivative of analytic function.

(c) State and prove Liouville's theorem and using this theorem prove that every polynomial equation of degree \( n \) has \( n \) roots.

Attempt any four parts of the following:

(a) Expand the function \( f(z) = \tan^{-1} z \) in powers of \( z \).

(b) Define the singularity of a function. Find the singularity (ties) of the functions

(i) \( f(z) = \sin \frac{1}{z} \)

(ii) \( g(z) = \frac{e^z}{z^2} \)

(c) Evaluate \( \int_{-\infty}^{\infty} \frac{x^2 \, dx}{(x^2 + 1)^2 \left( x^2 + 2x + 2 \right)} \).

(d) Evaluate \( \int_{0}^{2\pi} \frac{d\theta}{a + b \sin \theta} \) if \( a > |b| \).
(e) Evaluate \( \int_{-\infty}^{\infty} \frac{x \sin \pi x}{x^2 + 2x + 5} \, dx. \)

(f) Define the conformal mapping. Prove that an analytic function \( f(z) \) ceases to be conformal at the points \( z_0 \), where \( f'(z_0) = 0. \)

Attempt any two of the following: \( 10 \times 2 = 20 \)

(a) Define the coefficients of skewness and kurtosis. Compute the coefficient of skewness from the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>13</td>
<td>8</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

(b) Define the coefficients of regression and correlation. Calculate the coefficient of correlation between the marks obtained by 8 students in Mathematics and Statistics:

<table>
<thead>
<tr>
<th>Students</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>25</td>
<td>30</td>
<td>32</td>
<td>35</td>
<td>37</td>
<td>40</td>
<td>42</td>
<td>45</td>
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<tr>
<td>Statistics</td>
<td>08</td>
<td>10</td>
<td>15</td>
<td>17</td>
<td>20</td>
<td>23</td>
<td>24</td>
<td>25</td>
</tr>
</tbody>
</table>

(c) Define the binomial distribution and obtain the expression for the Poisson distribution as a limiting case of binomial distribution.
5. Attempt any two of the following: 10x2=20
(a) Find the roots of the cubic equation
\[ ax^3 + 3bx^2 + 3cx + d = 0 \]
using Cardan's method.
(b) Fit a parabola \( y = ax^2 + bx + c \) to the following data taking \( x \) as independent variable:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>19</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>11</td>
<td>13</td>
<td>17</td>
<td>19</td>
<td>23</td>
<td>29</td>
</tr>
</tbody>
</table>

(c) The regression lines of \( y \) on \( x \) and of \( x \) on \( y \) are respectively \( y = ax + b \) and \( x = cy + d \). Show that the means are \( \bar{x} = (bc + d)(1-ac) \) and \( \bar{y} = (ad + b)(1-ac) \) and correlation coefficient between \( x \) and \( y \) is \( \sqrt{ac} \). Also, show that the ratio of the standard deviations of \( y \) and \( x \) is \( \sqrt{a/c} \).