B. Tech.

(SEM. IV) THEORY EXAMINATION 2011-12

THEORY OF AUTOMATA AND FORMAL LANGUAGES

Time : 3 Hours Total Marks : 100

Note : (1) Attempt all questions.
(2) All questions carry equal marks.
(3) Notations/Symbols/Abbreviations used have usual meanings.
(4) Make suitable assumption(s), wherever required.

1. Attempt any two parts of the following:

(a) Obtain deterministic finite automata (DFA) with minimum number of states which accepts the same language which is accepted by the following nondeterministic finite automata (NFA).

(b) (i) Draw finite automata recognizing the following language:

\[ 1(1 + 10)^* + 10(0 + 01)^* \]
(ii) Determine whether complement of a nonregular language is also nonregular. Prove your answer.

(c) (i) Simplify the regular expression \((r(r + s)^*)^*\).

(ii) Draw the finite automata which accepts all the strings containing both 11 and 010 as substrings.

2. Attempt any two parts of the following:

(a) State pumping lemma for regular languages. Use pumping lemma to prove that the language \(L\), defined as follows, is not regular.

\[ L = \{0^m 1^n \mid m \text{ and } n \text{ are positive integers and } m \neq n\} \]

(b) (i) Explain the difference between Moore machine and Mealy machine. Describe with the help of an example how a Moore machine can be converted to a Mealy machine.

(ii) Design a finite automata which accepts the complement of the language accepted by the following automata:

(c) Obtain the regular expression corresponding to the following automata:
3. Attempt any two parts of the following:
   (a) (i) Consider the grammar G given as follows:
         \[ S \rightarrow AB \mid aaB \]
         \[ A \rightarrow a \mid Aa \]
         \[ B \rightarrow b \]
         Determine whether the grammar G is ambiguous or not. If G is ambiguous then construct an unambiguous grammar equivalent to G.
   (ii) The family of context free languages is closed under star-closure but is not closed under difference.

   (b) (i) Given a context free Grammar G. Write an algorithm (if it exists) to determine whether \( L(G) \) is infinite or not.
   (ii) Given the following CFG having S as start symbol, find an equivalent CFG with no useless symbols:
         \[ S \rightarrow aAa \mid bBb \mid \epsilon \]
         \[ A \rightarrow C \mid a \]
         \[ B \rightarrow C \mid b \]
         \[ C \rightarrow CDE \mid \epsilon \]
         \[ D \rightarrow A \mid B \mid ab \]

   (c) What is difference between Chomsky normal form (CNF) and Greibach normal form (GNF)? Convert the following grammar in Greibach normal form:
         \[ S \rightarrow AB \]
         \[ A \rightarrow BS \mid BB \mid b \]
         \[ B \rightarrow a \mid aAb \]

4. Attempt any two parts of the following:
   (a) Construct a PDA that accepts the language \( L \) over \( \{0, 1\} \) by empty stack which accepts all the strings of 0’s and 1’s in which number of 0’s are twice of the number of 1’s.

   (b) Convert the given PDA M to equivalent context free grammar. The PDA M is defined as M \( (\{q_0, q_1\}, \{0, 1\}, \{X, Z_0\}, \delta, q_0, Z_0, \{q_1\}) \) where \( \delta \) is given as follows:
       \[ \delta (q_0, 1, Z_0) = (q_0, XZ_0) \]
\[ \delta(q_0, 1, X) = (q_0, XX) \]
\[ \delta(q_0, 0, X) = (q_0, X) \]
\[ \delta(q_0, \epsilon, X) = (q_1, \epsilon) \]
\[ \delta(q_1, \epsilon, X) = (q_1, \epsilon) \]
\[ \delta(q_1, 0, X) = (q_1, XX) \]
\[ \delta(q_1, 0, Z_0) = (q_1, \epsilon) \]

(c) (i) Define a deterministic push down automata (DPDA). Write a DPDA which accepts the language \( L = \{a^n b^m c^n | n \text{ and } m \text{ are arbitrary positive integers} \} \).

(ii) Prove that every language accepted by a PDA by final state is also accepted by some PDA by empty stack.

5. Attempt any two parts of the following:

(a) What do you understand by Instantaneous Description of a Turing Machine? Design a Turing machine that computes the integer function \( f \) which multiplies two given positive integers defined as follows:
   \[ f(m, n) = m \times n. \]

(b) (i) Let \( A = \{001, 0011, 11, 101\} \) and \( B = \{01, 111, 111, 010\} \). Does the pair \((A, B)\) have Post Correspondence (PC) solution? Does the pair \((A, B)\) have Modified Post Correspondence (MPC) solution?

(ii) Prove that recursively enumerable languages are closed under intersection.

(c) What do you understand by undecidable problem? State the Halting Problem and prove that Halting problem is undecidable.